GEOMETRY AND APPLICATIONS ONLINE

CELEBRATING THE 80TH BIRTHDAY OF

DMITRI ALEKSEEVSKY

7-9 SEPTEMBER 2020

–PROGRAMME–

Program of planned talks

time	1st day	2nd day	3rd day
13:10 -14:10	José Figueroa-O'Farrill	Anna Fino	Giovanna Citti
14:20 - 15:20	Jorge Lauret	Christoph Böhm	Simon Salamon
15:30 - 16:30	Ivan Izmestiev	Remco Duits	Jaroslav Hrdina
16:40 - 17:40	Vicente Cortés	Alessandro Sarti	Antonio J. Di Scala

Titles of Talks & Abstracts

Talks on September 7th

José Figueroa-O'Farrill (University of Edinburgh)

- *Title of Talk:* **Kinematical G-structures and their intrinsic torsion**
- *Abstract:* When Dmitri was in his 20s, Bacry and Lévy-Leblond asked the question: *What are the possible kinematical symmetries of space and time?* and proceeded to give an answer by classifying kinematical Lie algebras subject to some "by no means compelling" assumptions, which were removed by Bacry and Nuyts when Dmitri was in his 40s. Taking this approach to their logical conclusion, when Dmitri was in his (late) 70s, Stefan Prohazka and I classified simply-connected kinematical Klein geometries and found that modulo some low-dimensional exceptions, they fall into one of several types: lorentzian, galilean (a.k.a.. Newton-Cartan), carrollian and aristotelian. In this talk, I will introduce these kinematical geometries as G-structures and refine their classification using their intrinsic torsion.

Jorge Lauret (Universidad Nacional de Córdoba)

- *Title of Talk:* **The Lichnerowicz Laplacian and Prescribed Ricci problem on homogeneous manifolds**
- *Abstract:* Let M be a homogeneous manifold and let G be any of the Lie groups acting transitively on M. After an overview of potential applications, we will give a formula for the differential $dRic$ of the function Ric at a G-invariant metric g on M, which is precisely the Lichnerowicz Laplacian acting on G-invariant symmetric 2-tensors when G is compact. The formula is in terms of the moment map for the variety of Lie algebras.

  Some applications obtained to the classical Prescribed Ricci problem (i.e., given a symmetric 2-tensor T on a manifold M, one asks about the existence and uniqueness of a metric g on M and a positive constant c such that $Ric(g) = cT$) will also be given.

  This is joint work in progress with Cynthia Will.

Ivan Izmestiev (TU Wien)

- *Title of Talk:* **Discrete Lipschitz-Killing curvatures**
- *Abstract:* Weyl's tube formula says that the volume of a t-neighborhood of a Riemannian manifold isometrically embedded into a Euclidean space is a polynomial in t, moreover the coefficients of this polynomial are independent of the embedding. This gives rise to a family of intrinsic quantities called the *Lipschitz-Killing curvatures* of a Riemannian manifold. In this talk we review a parallel story in the discrete case, where instead of a Riemannian manifold one considers a manifold glued from Euclidean polyhedra. Following the works of Regge, Cheeger–Mueller–Schrader, P. McMullen, and Budah, we describe the discrete total scalar curvature and the discrete Chern-Gauss-Bonnet theorem.

Vicente Cortés (Universität Hamburg)

- *Title of Talk:* **Quaternionic Kähler manifolds of cohomogeneity one**
- *Abstract:* By results of Dancer-Swann and Podestà-Verdiani there are no complete quaternionic Kähler manifolds of positive scalar curvature for which the isometry group is of cohomogeneity one. We construct complete quaternionic Kähler manifolds of cohomogeneity one of negative scalar curvature. Our methods are twofold: 1) deformation of homogeneous spaces (Alekseevsky spaces) by the HK/QK-correspondence and 2) construction from projective special real manifolds of cohomogeneity one.
  1) is based on joint work with Saha and Thung (arXiv: https://arxiv.org/abs/2001.10026, https://arxiv.org/abs/2001.10032),
  2) is based on joint work with Dyckmanns, Jüngling, and Lindemann (arXiv: https://arxiv.org/abs/1701.07882, Asian J. Math. to appear).

Talks on September 8th

Anna Fino (Università di Torino)

- *Title of Talk:* **An overview of G_2-structures and $Spin(7)$-structures on homogeneous spaces**
- *Abstract:* In this talk we review known results on invariant G_2-structures and $Spin(7)$-structures on homogeneous spaces. In particular, we describe a recent classification (joint with D. Alekseevsky, I. Chrysikos and A. Raffero) on simply connected compact homogeneous spaces admitting invariant $Spin(7)$-structures (arXiv: https://arxiv.org/abs/1904.00643, Int. J. Math. to appear).

Christoph Böhm (WWU Münster)

- *Title of Talk:* **Homogeneous Einstein metrics on Euclidean spaces are Einstein solvmanifolds**
- *Abstract:* We show that homogeneous Einstein metrics on Euclidean spaces are Einstein solvmanifolds, using that they admit periodic, integrally minimal foliations by homogeneous hypersurfaces. For the geometric flow induced by the orbit-Einstein condition, we construct a Lyapunov function based on curvature estimates which come from real GIT. This is joint work with Ramiro Lafuente.

Remco Duits (Eindhoven University of Technology)

- *Title of Talk:* **PDEs on the Homogeneous Space of Positions and Orientations**
- *Abstract:* We solve and analyze PDEs on the homogeneous space of positions and orientations. This homogeneous space is given by $\mathbb{M} = SE(d)/H$ where $SE(d)$ is the roto-translation Lie group and $H \equiv SO(d-1)$ the subgroup of rotations around a reference axis. We consider $d \in \{2, 3\}$.

We solve the following PDEs on $\mathbb{M}$ **analytically**:
- Degenerate and non-degenerate (convection-)diffusion systems on $\mathbb{M}$, cf. [1]
- Forward Kolmogorov PDEs of α-stable Lévy processes on $\mathbb{M}$, cf. [2].

this is done by a Fourier transform on $\mathbb{M}$, cf. [2].

We solve the following PDEs on $\mathbb{M}$ **numerically**:
- Nonlinear Diffusions on $\mathbb{M}$, cf. [3],
- Mean Curvature Flows and Total Variation Flows on $\mathbb{M}$, cf. [4] ($d = 2, 3$), [5, 6] ($d = 2$),
- Eikonal PDEs for sub-Riemannian and Finslerian geodesic front propagation on $\mathbb{M}$, cf. [7, 8],

  with new data-driven versions (improving the underlying Cartan connections [16, 17]).

via anisotropic fast-marching [9], left-invariant finite difference techniques [10] or Monte-Carlo simulations [2] of the underlying SDEs. The numerics is tested to our new exact solutions of the PDEs [2, 11] and of the sub-Riemannian geodesics in $\mathbb{M}$ [12].

We show their applications in medical image analysis in enhancement of fibers/blood vessels in 2D and 3D medical images [3, 4], and in fiber-enhancement [13], denoising [4], fiber-tracking [14], and structural connectivity quantification [15] in DW-MRI.

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Alessandro Sarti (CAMS - Centre d'Analyse et de Mathématique sociales)

- *Title of Talk:* **Heterogeneous neurogeometry**
- *Abstract:* I will remind the concept of neurogeometry and the construction of the Citti-Sarti model of the primary visual cortex (V1) as a fiber bundle in the SE(2) group equipped with a sub-Riemannian metric. The same approach has been used to model different kind of cells of V1 associated to different Lie groups. In this talk I will consider a model of heterogeneous neurogeometry allowing the interaction of different Lie groups.

Talks on September 9th

Giovanna Citti (Università di Bolognia)

- *Title of Talk:* **Perceptual completion and submanifolds of fixed degree in Lie groups**
- *Abstract:* As it is well know, the visual cortex can be described as a Lie group with a sub-Riemannian metric. I'll present a model, joint work with A. Sarti, where subjective surfaces are described as $2D$ subriemannian minimal surfaces in an Engel-type group. Subriemannian hypersurfaces are very well understood, but very little in known for higher codimension. In a joint work with Giovannardi and Ritoré we study surfaces of arbitrary degree in Lie groups, and discover the existence of abnormal, isolated surfaces, which is the exact analogous of the well known phenomena of abnormal geodesics.

Simon Salamon (King's College London)

- *Title of Talk:* **Quotients of $\mathbb{R}^8$ and reduced holonomy**
- *Abstract:* The action of a 2-torus on $\mathbb{R}^8$ leads one to ask what happens to the $SO(5)$-invariant metric with holonomy G_2 on a cone over $\mathbb{C}P^3$ when one quotients by $U(1)$. This is a problem motivated by String Theory. I shall explain the underlying twistor geometry, how one represents points of the quotient by bivectors, and describe the induced symplectic structure on $\mathbb{R}^6$. This is joint work with Bobby Acharya and Robert Bryant.

Jaroslav Hrdina (Brno University of Technology)

- *Title of Talk:* **Clifford algebras and engineering applications**
- *Abstract:* We present a panorama of Clifford algebras as conformal, projective and euclidean ones. We discuss their benefits and disbenefits for design and control robotics based mechanisms. Mainly, we focus on control of industry manipulators, and camera systems. The problems have been motivated by concrete engineering tasks.

Antonio J. Di Scala (Politecnico di Torino)

- *Title of Talk:* **The normal holonomy group of complex submanifolds**
- *Abstract:* This talk is going to be a survey talk of results, ideas and questions about the normal holonomy group of complex submanifolds of complex space forms.